
Student Number

Barker College

2020 TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

Staff Involved:

- ARP*
- LMD
- RJW
- RAS
- AYG
- ESP
- AXD
- ALY
- JZT

Friday 14th August 2020
8:30 AM

145 copies

General Instructions:

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A separate reference sheet is provided
- For questions in Section II, show relevant mathematical reasoning and / or calculations

Total marks:
100

Section I – 10 marks (pages 2 - 6)

- Attempt Multiple Choice Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 7 - 25)

- Attempt Questions 11 - 19
- Show all necessary working
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

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1. What is the derivative of $\ln(\cos x)$?

   (A) $-\sec x$

   (B) $-\tan x$

   (C) $\sec x$

   (D) $\tan x$

2. The mean of a set of 10 scores is 14. Another two scores are included and the new mean is 16. What is the mean of the two additional scores?

   (A) 4

   (B) 16

   (C) 18

   (D) 26

3. What is the greatest value of the function $f(x) = -2\cos x + 3$?

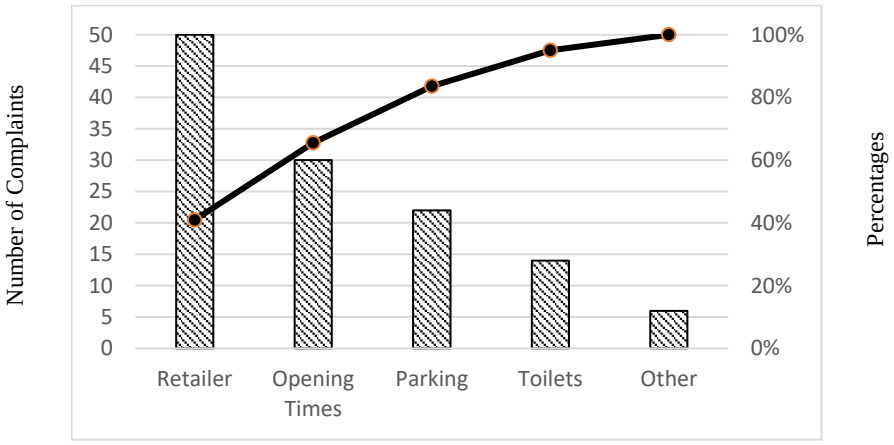
   (A) 1

   (B) 3

   (C) 5

   (D) 7

4. Staff at a large department store wanted to determine the key complaints customers had over a period of one month. The following Pareto chart was prepared.



Approximately what percentage of complaints were **Opening Times**?

(A) 12%

(B) 25%

(C) 30%

(D) 60%

5. The function $f(x) = x^2 - 5x + 1$ is reflected about the $y-$ axis. What is the equation of the new function?

(A) $f(x) = x^2 + 5x + 1$

(B) $f(x) = x^2 + 5x - 1$

(C) $f(x) = -x^2 + 5x + 1$

(D) $f(x) = -x^2 + 5x - 1$

6. Using the trapezoidal rule with 4 subintervals, which expression gives the approximate area under the curve $y = xe^x$ between $x = 1$ and $x = 3$?

(A) $\frac{1}{4}(e^1 + 6e^{1.5} + 4e^2 + 10e^{2.5} + 3e^3)$

(B) $\frac{1}{4}(e^1 + 3e^{1.5} + 4e^2 + 5e^{2.5} + 3e^3)$

(C) $\frac{1}{2}(e^1 + 6e^{1.5} + 4e^2 + 10e^{2.5} + 3e^3)$

(D) $\frac{1}{2}(e^1 + 3e^{1.5} + 4e^2 + 5e^{2.5} + 3e^3)$

7. In a Year 12 drama class of 26 students, all students do either music or art or both. 11 do music. 5 do both music and art. What is the probability that a randomly chosen student does music and art, given they do art?

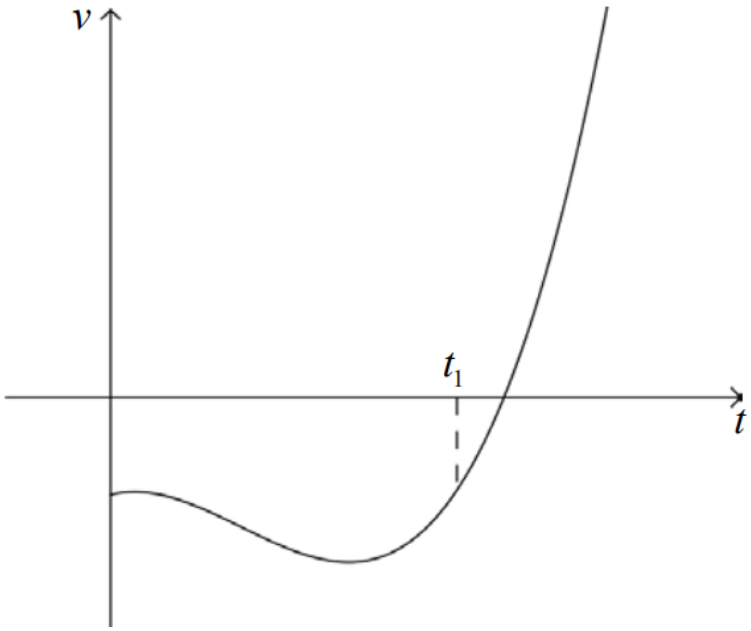
(A) $\frac{5}{26}$

(B) $\frac{1}{4}$

(C) $\frac{1}{3}$

(D) $\frac{5}{11}$

8. The graph below shows the velocity v of a particle moving in a straight line as a function of time t. The positive direction of the motion is to the right.



Which statement describes the motion of the particle when $t = t_1$?

(A) The velocity is positive, and acceleration is positive

(B) The velocity is negative, and the acceleration is positive

(C) The velocity is positive, and the particle is to the right of its initial position

(D) The velocity is negative, and the particle is to the right of its initial position

9. The first three terms of a geometric series are $\frac{1}{x}+\frac{1}{x^2}+\frac{1}{x^3}+\ldots$.

   If the series has a limiting sum, then which of the below is true?

   (A) $x < -1$ or $x > 1$

   (B) $-1 < x < 1$

   (C) $|x| < 1$

   (D) None of the above

10. The function $y = f(x)$ has a turning point at $(6,2)$. It is transformed to $y = 3f(-2x + 4) + 2$. Where is the new turning point?

   (A) $(-1,8)$

   (B) $(-7,8)$

   (C) $(-10,8)$

   (D) $(-16,8)$

End of Section I

Section II

90 marks
Attempt Questions 11 - 19
Allow about 2 hours and 45 minutes for this section

In Questions 11–19, your responses should include relevant mathematical reasoning.

Question 11 (10 marks)

(a) Differentiate the following functions.

(α) $(3x + 4)^5$. **1**

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(β) $\dfrac{\tan x}{x}$. **1**

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(b) (i) Differentiate $y = xe^{3x}$. **1**

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(ii) Hence find the exact value of $\int_0^2 e^{3x}(3+9x)\,dx$. **2**

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(c) A balloon drifts 100 km from point A to point B on a bearing of 28°. At point B the balloon changes direction and drifts 50 km to point C on a bearing of 114°.

(i) Draw a diagram showing the above information. **1**

(ii) Show that the distance from point A to point C is 115 km to the nearest kilometre. **2**

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(iii) Find the bearing of point A from point C. Give your answer to the nearest degree. **2**

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Question 12 (10 marks)

Evaluate the following integrals:

(a) $\int 3^x \, dx.$ **1**

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(b) $\int \frac{x}{x^2+3} \, dx.$ **2**

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(c) $\int_0^{\frac{\pi}{2}} \sin \frac{x}{2} \, dx.$ **2**

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(d) During a storm, water flows into a 7000 litre tank at a rate of $\frac{dv}{dt}$ litres per minute, where $\frac{dv}{dt} = 120 + 26t - t^2$ and t is the time in minutes since the storm began.

(i) At what times is the tank filling at twice the initial rate? **2**

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(ii) Initially the tank contains 1500 litres of water. When the storm finishes, 30 minutes after it began, the tank is overflowing. **3**

How many litres of water have been lost?

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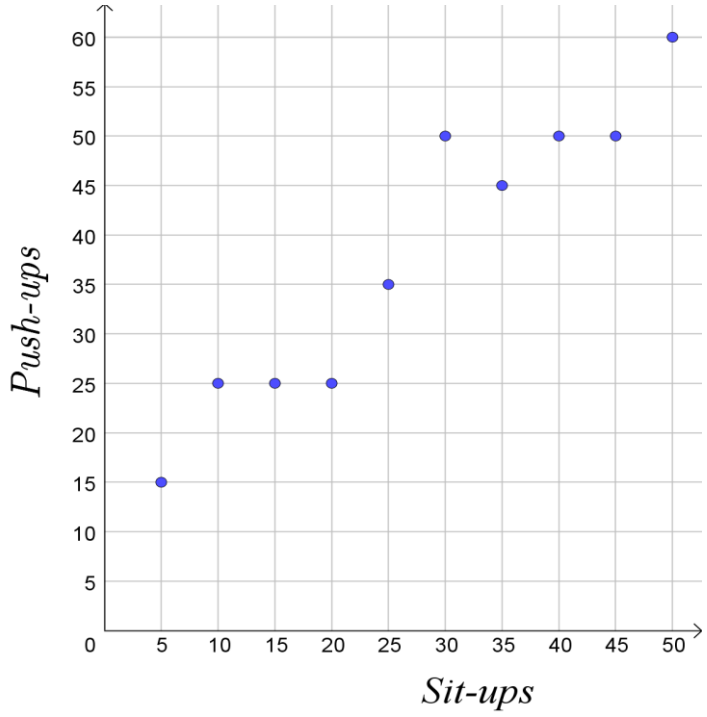
Question 13 (10 marks)

(a) The scatterplot below shows the number of Push-ups (P) and the number of Sit-ups (S) performed by 10 students during a fitness test.



(i) Calculate the value of the correlation coefficient correct to 2 decimal places. Comment on the strength and direction of correlation. **2**

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(ii) Determine the equation of the least-squares regression line for this data in terms of Push-ups (P) and Sit-ups (S). **2**

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(iii) Predict the number of sit-ups a person can do if they can do 100 push-ups. **1**

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(b) An infinite geometric series has a first term of 8 and a limiting sum of 12.

(i) Calculate the common ratio. **1**

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(ii) Hence, calculate the sum of the first three terms. **1**

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(c) In an arithmetic series, the sum of the first 16 terms is 288 and the sixth term is 8.

Find the first term and the common difference. **3**

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Question 14 (10 marks)

(a) A table of future value interest factors for an annuity of \$1 is shown below.

Period	Interest rate per period				
	2.5%	3%	3.5%	4%	4.5%
1	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0250	2.0300	2.0350	2.0400	2.0450
3	3.0756	3.0909	3.1062	3.1216	3.1370
4	4.1525	4.1836	4.2149	4.2465	4.2782
5	5.2563	5.3091	5.3625	5.4163	5.4707
6	6.3877	6.4684	6.5502	6.6330	6.7169

(i) Jenna is saving for a holiday by contributing \$500 every six months into an annuity that pays interest at the rate of 9% p.a., compounded every six months. Use the table above to determine how much she will have in 3 years time. **1**

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(ii) Toby is saving for the same trip but needs \$5000 in total. How much more than Jenna does he need to contribute every six months, using the same interest rate, if he wishes to have enough money in 3 years time? **2**

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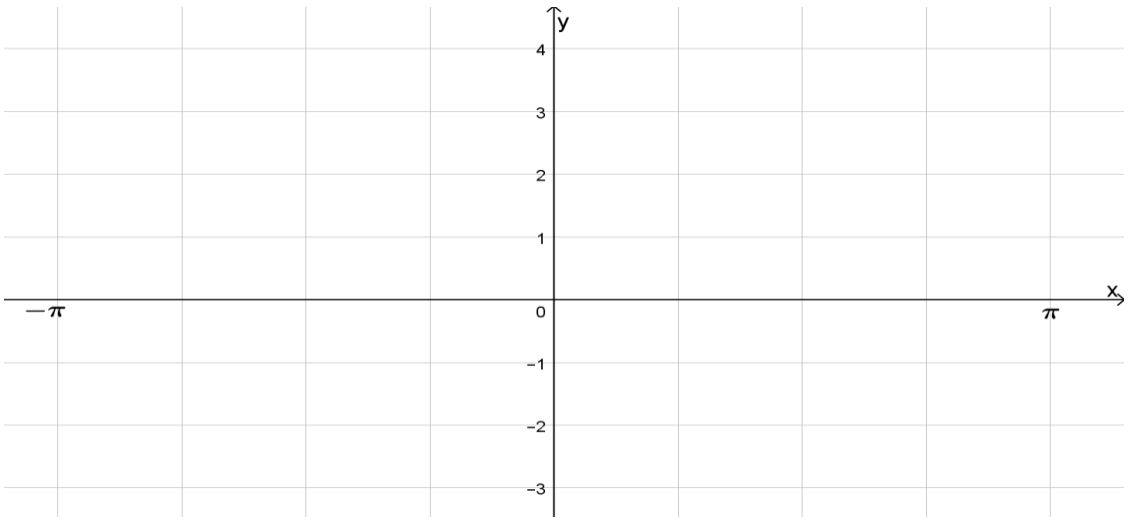
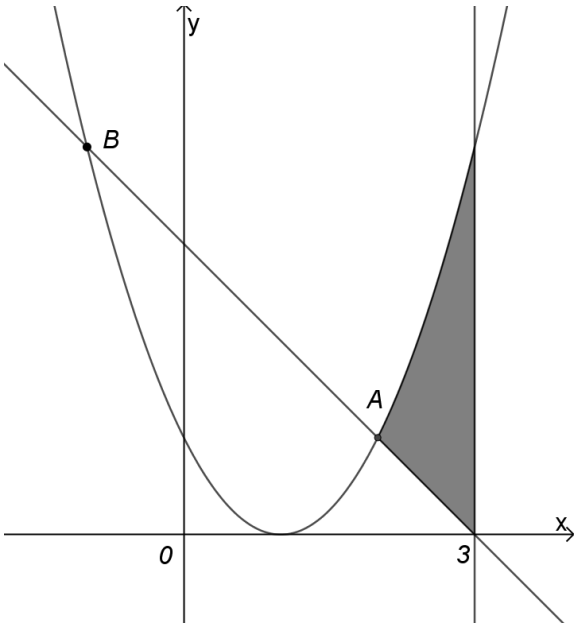
(b) On the axes below sketch the function $f(x) = 2\sin 2x + 1$ in the domain $-\pi \leq x \leq \pi$. (You do not need to show x −intercepts). **3**



(c) The diagram below shows the curve $y = (x-1)^2$ intersecting with the line $x + y = 3$ at the points A and B.



(i) Verify that the coordinates of A are $(2,1)$. **1**

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(ii) Hence find the shaded area enclosed by the curve $y = (x-1)^2$ and the lines $x + y = 3$ and $x = 3$. **3**

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Question 15 (10 marks)

(a) Let $f(x) = 3x^4 - 8x^3 + 6$.

(i) Find the coordinates of the stationary points of $y = f(x)$, and determine their nature. **4**

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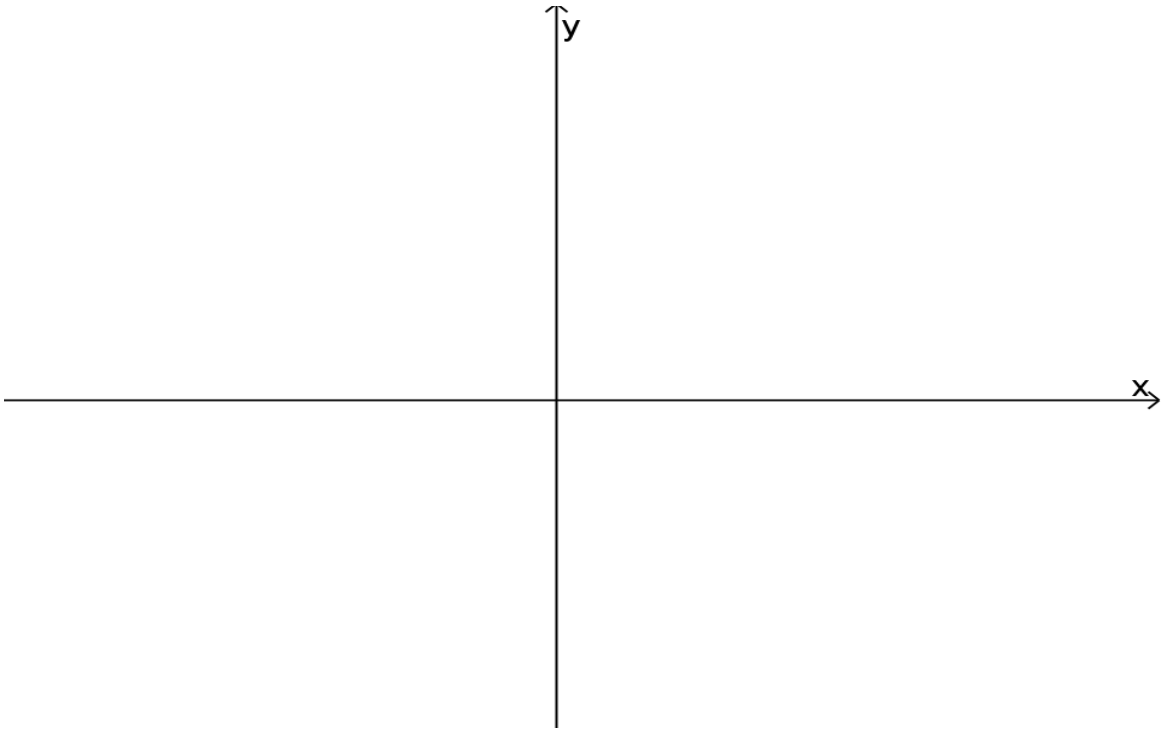
(ii) Hence, sketch the graph of $y = f(x)$ showing all stationary points. **2**



(b) For the probability distribution below $E(X) = 3.34$.

x	1	3	a	6
$p(x)$	0.4	0.12	0.3	b

(i) Evaluate a and b. **2**

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(ii) Evaluate the Variance. **2**

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Question 16 (10 marks)

(a) Due to overfishing in a particular bay, the number (N) of a particular species of fish is dropping exponentially according to the formula $\frac{dN}{dt} = -kN$, where time (t) is measured in years after 1930. It is known that in 1930 there were $25{,}000$ fish of this species and by 2020 there were only 2000 fish.

(i) Show that $N = Ae^{-kt}$, where A and k are constants, satisfies the equation $\frac{dN}{dt} = -kN$. **1**

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(ii) Find the value of A and show that $k \div 0.028$. **2**

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(iii) This species of fish will be declared extinct in this bay when the number drops below 500 fish. In which year will this occur? **2**

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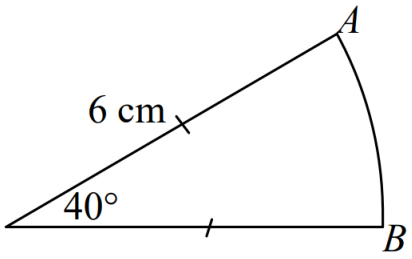
(b) The angle of a sector in a circle of radius 6 cm is 40°, as shown in the diagram below. **2**



NOT TO SCALE

Find the exact length of arc AB.

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(c) Solve $2\cos^2 x - \cos x = 1$ for $0 \leq x \leq 2\pi$. **3**

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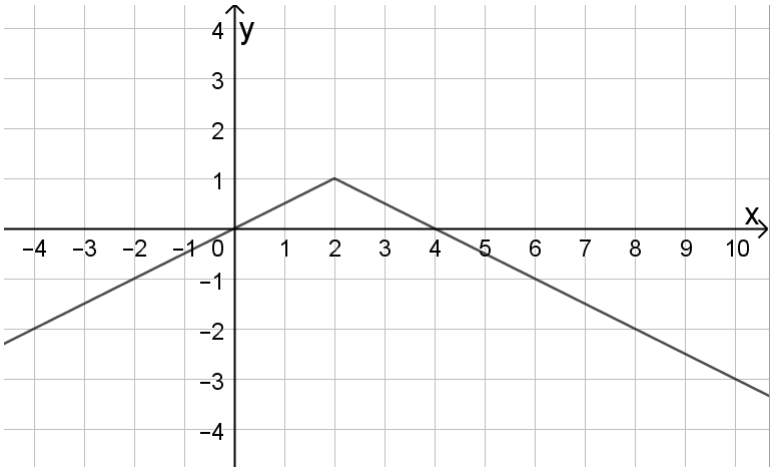
Question 17 (10 marks)

(a) The function $f(x) = |x|$ is transformed and the equation of the new function is of the form $y = kf(x + b) + c$ where k, b and c are constants. **3**

The graph of the new function is shown below.



What are the values of k, b and c?

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(b) (i) When Josephine started a new job, \$450 was deposited into her superannuation fund at the beginning of each month. The money was invested at 0.5% per month, compounded monthly. **2**
Let \$$P$ be the value of the investment after 300 months, when Josephine retires.

Show that $P = \$313\ 406.52$.

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Question 17 (b) continues on the following page

(ii) After retiring, Josephine withdraws \$2500 from the account at the end of each month, without making any further deposits. The account continues to earn interest at 0.5% per month.
Let $\$A_n$ be the amount left in the account n months after Josephine's retirement.

(α) Show that $A_n = (P - 500\,000) \times 1.005^n + 500\,000$ where P was defined in (i). **3**

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(β) For how many months after retirement will there be money left in the account? **2**

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Question 18 (10 marks)

(a) A particle is moving in a straight line with velocity $v = 3e^t + 6e^{-t}$.
It begins its motion at the Origin O, t is in seconds and v is metres per second.

(i) Find the displacement function, x, of the particle, at the time t seconds. **2**

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(ii) Find the exact time when the particle is at $x = 10$ metres. **3**

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(b) The function $y = e^x$ is reflected about the y −axis and moved up by 1 unit to give the function $f(x)$.

(i) Write down the equation of the function $f(x)$. **1**

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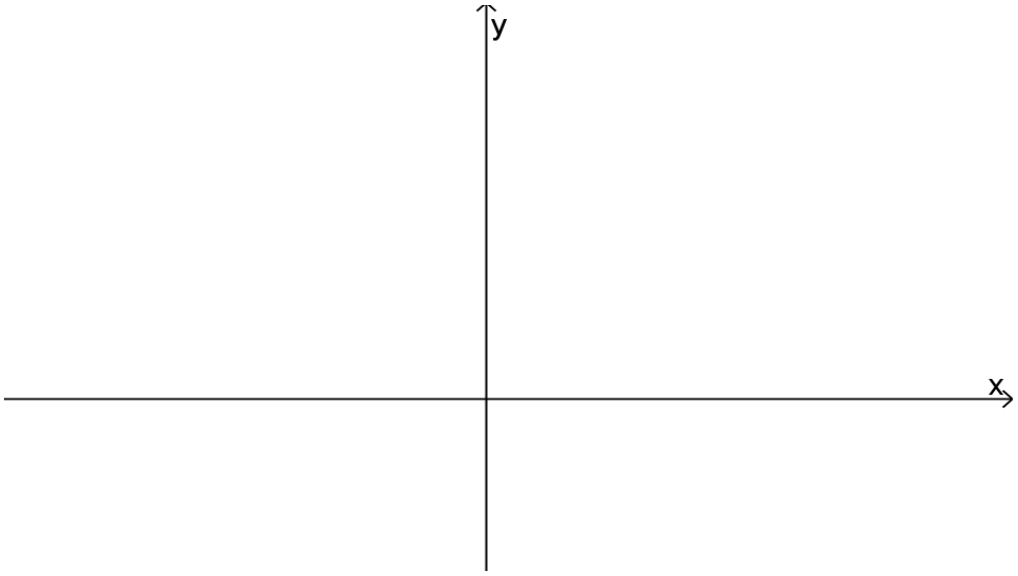
(ii) Sketch $y = f(x)$ on the number plane below showing all important details. **1**



(iii) Consider the function $g(x) = \ln x$. **3**

Determine the composite function $f\big(g(x)\big)$ indicating its domain and range.

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Question 19 (10 marks)

(a) A student tosses two regular six-sided dice and writes down the two numbers showing on the uppermost faces.

(i) Find the probability that their sum is greater than 7 and an even number. **2**

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(ii) Find the probability that their sum is greater than 7 and an even number, given that at least one of the numbers is even. **2**

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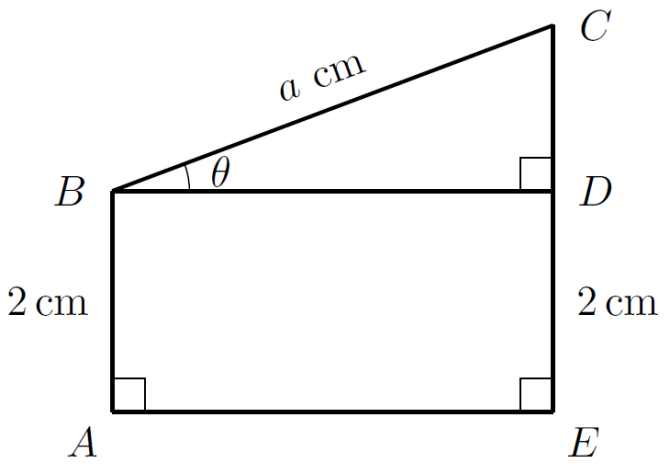
(b) The figure shown represents a wire frame made up of 6 parts; AB, BC, CD, ED, AE and BD. $AB = ED = 2\ cm$ and $BC = a\ cm$ where $a > 0$.

$\angle CBD = \theta$, *where* $0 < \theta < \frac{\pi}{2}$



NOT TO SCALE

(i) Find BD and CD in terms of a and θ. **1**

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(ii) Find the length L of the wire frame in terms of a and θ. **1**

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Question 19 (b) continues on the following page

(iii) Given that $a = 3\sqrt{5}$ find the exact maximum possible length of wire in the frame. **4**

END OF PAPER

MULTIPLE CHOICE

1. $y = \ln(\cos x)$ $\quad y' = \dfrac{-\sin x}{\cos x}$

$= -\tan x$ **(B)**

2. $14 \times 10 = 140$

$16 \times 12 = 192$

DIFF $= 52$ $\quad \bar{x} = 26$ **(D)**

3. WHEN $x = -1$

$f(-1) = -2 \times -1 + 3$

$= 5$ **(C)**

4. APPROX $65\% - 40\% = 25\%$ **(B)**

5. REFLECT ABOUT y:

$f(-x) = (-x)^2 - 5(-x) + 1$

$= x^2 + 5x + 1$ **(A)**

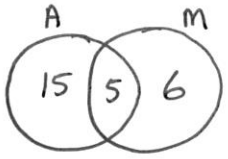
6. $h = \dfrac{3-1}{4} = \dfrac{2}{4} = \dfrac{1}{2}$

x	1	1.5	2	2.5	3
y_n	e^1	$1.5e^{1.5}$	$2e^2$	$2.5e^{2.5}$	$3e^3$
	y_0	y_1	y_2	y_3	y_4

$A \approx \dfrac{1/2}{2}\left[e^1 + 3e^3 + 2\left(1.5e^{1.5} + 2e^2 + 2.5e^{2.5}\right)\right]$

$= \dfrac{1}{4}\left(e^1 + 3e^{1.5} + 4e^2 + 5e^{2.5} + 3e^3\right)$ **(B)**

7.



$\therefore \ 5/20 = 1/4$ **(B)**

8. · V IS $-$VE ... BELOW x-AXIS

· a IS +VE ... +VE GRADIENT **(B)**

9. $r = \dfrac{1}{x^2} \div \dfrac{1}{x} = \dfrac{1}{x}$

$\therefore \ -1 < \dfrac{1}{x} < 1$

WHICH MEANS $x < -1$ or $x > 1$ **(A.)**

10. $y = 3f(-2[x-2]) + 2$

VERT. DILATE ③ | HORIZ. DILATE ① | RIGHT 2 ② | UP 2 ④

x COORD: $6 \times -1/2 = -3$ ①

$-3 + 2 = -1$ ②

y COORD: $2 \times 3 = 6$ ③

$6 + 2 = 8$ ④

$\therefore (-1, 8)$ **(A.)**

QUESTION 11.

a. **α.** $y = (3x+4)^5$

$y' = 5.3.(3x+4)^4 = 15(3x+4)^4$

β. $y = \dfrac{\tan x}{x}$ $\quad u = \tan x$ $\quad v = x$

$u' = \sec^2 x$ $\quad v' = 1$

$y' = \dfrac{x\sec^2 x - \tan x}{x^2}$

b. **i.** $y = xe^{3x}$ $\quad u = x$ $\quad v = e^{3x}$

$u' = 1$ $\quad v' = 3e^{3x}$

$y' = 3xe^{3x} + e^{3x}$

$= e^{3x}(3x+1)$

ii. $\int_0^2 e^{3x}(3+9x)\,dx$

$= \int_0^2 3e^{3x}(3x+1)\,dx$

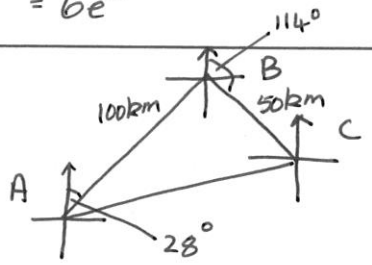
$= 3\int_0^2 e^{x}(3x+1)\,dx$

$= 3\left[x.e^{3x}\right]_0^2$

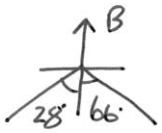
$= 3(2e^6 - 0)$

$= 6e^6$

c.i



ii.

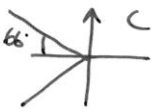


$x^2 = 100^2 + 50^2 - 2.100.50.\cos 94°$

$= 114.88...$

$= 115\ km.$

iii.



$\angle BCA = \frac{\sin 94°}{115} = \frac{\sin x}{100}$

$\sin x = \frac{100 \sin 94°}{115}$

$= 60.1°$

BEARING $= 360° - 66° - 60.1°$

$= 233°50' = 234°$

QUESTION 12

a. $\int 3^x dx = \frac{1}{\ln 3}.3^x + C.$

b. $\int \frac{x}{x^2+3}dx = \frac{1}{2}\int \frac{2x}{x^2+3}dx$

$= \frac{1}{2}\ln(x^2+3) + C$

c. $\int_0^{\pi/2} \sin\frac{x}{2}\,dx = \left[-2\cos\frac{x}{2}\right]_0^{\pi/2}$

$= \left(-2\cos\frac{\pi}{4}\right) - \left(-2\cos 0\right)$

$= -\frac{2}{\sqrt{2}} + 2$

RATIONALISE ETC.. $= 2 - \sqrt{2}$ OR $0.587...$

di. INITIAL RATE:

$t=0 \quad \frac{dV}{dt} = 120 + 0 + 0$

$= 120\ L/MIN$

$\therefore 120 \times 2 = 240$

$\therefore 240 = 120 + 26t - t^2$

$t^2 - 26t + 120 = 0$

$(t-20)(t-6) = 0$

$\therefore t = 20\ MINS,\ 6\ MINS.$

ii. $V = 120t + 13t^2 - \frac{t^3}{3} + C$

$t=0,\ V=1500 \quad 1500 = 0 + 0 - 0 + C$

$\therefore C = 0$

EQ'N: $V = 120t + 13t^2 - \frac{t^3}{3} + 1500$

PTO →

$t = 30$

$V = 3600 + 11700 - 9000 + 1500$

$= 7800$

$7000 - 7800 = -800L$

ie : 800L OVERFLOWED.

QUESTION 13.

ai. PUT IN CALC... $r = 0.96$

STRONG, POSITIVE CORRELATION

ii. PUT IN CALC...

$A = 12$

$B = 0.945...$

$P = 0.95S + 12$

iii. $100 = 0.95 \times S + 12$

$\therefore S = 92.6$

$= 93$ SIT UPS

bi. $a = 8$ $S_\infty = 12$

$12 = \frac{8}{1-r}$

$12(1-r) = 8$

$12 - 12r = 8$

$-12r = -4$

$r = 1/3$

ii. $S_3 = ?$ $a = 8$ $r = 1/3$

$S_3 = \frac{8\left(\left(\frac{1}{3}\right)^3 - 1\right)}{\frac{1}{3} - 1}$

$= \frac{104}{9}$ ☺

c. $T_6 = 8 = a + 5d$ —— ①

$S_{16} = 288 = \frac{16}{2}[2a + 15d]$

$288 = 8(2a + 15d)$

$288 = 16a + 120d$ —— ②

① × 16: $128 = 16a + 80d$ ③

② - ③: $160 = 40d$

$\therefore d = 4$

SUB INTO ①: $8 = a + 20$

$\therefore a = -12$

QUESTION 14.

ai. $9\% \div 2 = 4.5\%$

$3 \times 2 = 6$ PERIODS

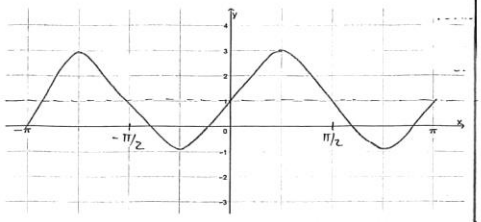
$\therefore \$500 \times 6.7169$

$= \$3358.45$

ii. $\$5000 \div 6.7169 = \$744.39$

$\$744.39 - \$500 = \$244.39$

b.



ci. SOLVE SIMULTANEOUSLY:

$(x-1)^2 = 3-x$

$x^2 - 2x + 1 = 3 - x$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, -1$

SUB. IN $x = 2$ TO $x+y=3$

$\therefore y = 1$

$\therefore (2,1)$ LIES ON BOTH.

ii. AREA = TOP − BOTTOM

$$A = \int_2^3 (x-1)^2 - (3-x)\,dx$$

$$= \int_2^3 x^2 - x - 2\,dx$$

$$= \left[\frac{x^3}{3} - \frac{x^2}{2} - 2x\right]_2^3$$

$$= (9 - 9/2 - 6) - (8/3 - 2 - 4)$$

$= 11/6$ UNITS^2.

QUESTION 15.

ai. $f'(x) = 12x^3 - 24x^2$

STAT PT: $0 = 12x^3 - 24x^2$

$0 = 12x^2(x-2)$

$\therefore x = 0, 2$

$f''(x) = 36x^2 - 48x$

$f''(0) = 0$ TEST IT!

x	-1	0	1
$f''(x)$	84	0	-12

HORIZ. $\therefore$ POI

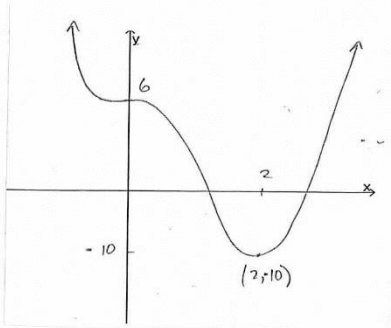
$f''(2) = 144 - 96$

$= 48$ ... +VE ∪

$\therefore$ MIN @ $(2, -10)$

HORIZ POI @ $(0, 6)$

ii.



bi.

x	1	3	a	6	sum
$p(x)$	0.4	0.12	0.3	b	1
$x\,p(x)$	0·4	0·36	0·3a	1·08	3·34 ← μ or $E(x)$
$x^2\,p(x)$	0·4	1·08	7·5	6·48	15·46 ← $E(x^2)$

$b = 1 - (0.4 + 0.12 + 0.3)$

$= 0.18$

$0.3a = 3.34 - (0.4 + 0.36 + 1.08)$

$0.3a = 1.5$

$\therefore a = 5$

ii. $\text{Var}(x) = E(x^2) - \mu^2$

$= 15.46 - 3.34^2$

$= 4.3044$

QUESTION 16.

i. $\frac{dN}{dt} = -k.Ae^{-kt}$

$= -kN$ as $N = Ae^{-kt}$.

ii. $t = 0 \quad 25000 = Ae^{0}$

$\therefore A = 25000$

$2000 = 25000e^{-90k}$

$0.08 = e^{-90k}$

$\frac{\ln(0.08)}{\ln(e)} = -90k$

$-2.52 = -90k$

$\therefore k = 0.0280\ldots$

$= 0.028$

iii. $500 = 25000e^{-0.028t}$

$0.02 = e^{-0.028t}$

$\frac{\ln(0.02)}{\ln(e)} = -0.028t$

$-3.91 = -0.028t$

$139.71 = t$

$\therefore t = 140$ YEARS

$\therefore$ DATE = 2070 AD.

b. OLD SCHOOL!

$\frac{40^\circ}{360^\circ} \times 2 \times \pi \times 6\text{cm}$

$= \frac{1}{9} \times 12\pi$

$= \frac{4\pi}{3}$ cm. (OR USE $\ell = r\theta$)

c. $2\cos^2 x - \cos x - 1 = 0$

Let $u = \cos x$

$2u^2 - u - 1 = 0$

$(2u+1)(u-1) = 0$

$\therefore u = -\frac{1}{2}, 1$

$\cos x = -\frac{1}{2}$ (USE ASTC)

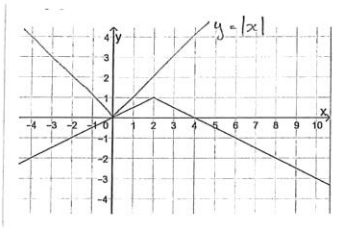
$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$\cos x = 1$ (USE GRAPH)

$\therefore x = 0, 2\pi$

QUESTION 17.

a.



- FLIPPED ABOUT x & "WIDENED" ... $k = -\frac{1}{2}$
- MOVED UP 1 ... $c = 1$
- MOVED RIGHT 2 ... $b = -2$

PTO

bi. ANNUITIES Q'N...

$A_1 = 450(1.005)^{300}$

$A_2 = 450(1.005)^{299}$

$\vdots$

$A_{300} = 450(1.005)^{1}$

$\therefore A_{TOTAL} = 450(1.005^{300} + 1.005^{299} + \ldots + 1.005)$

GP!

$S_{300} = \dfrac{1.005(1.005^{300} - 1)}{0.005}$

$a = 1.005$

$r = 1.005$

$n = 300$

$\therefore A_{TOTAL} = 450 \times 696.45\ldots$

$= \$313\,406.52$

ii. $A_1 = P(1.005) - 2500$

α. $A_2 = A_1(1.005) - 2500$

$= \{P(1.005) - 2500\} \times 1.005 - 2500$

$= P(1.005)^2 - 2500(1.005 + 1)$

$A_3 = P(1.005)^3 - 2500(1.005^2 + 1.005 + 1)$

...SHOW MORE WORKING THAN ABOVE. I AM LIMITED BY SPACE!

$A_n = P(1.005)^n - 2500(1.005^{n-1} + \ldots + 1)$

GP!

$a = 1$

$r = 1.005$ $\quad S_n = \dfrac{1(1.005^n - 1)}{0.005}$

$n = n$

$A_n = P(1.005)^n - \dfrac{2500(1.005^n - 1)}{0.005}$

$= P(1.005)^n - 500\,000(1.005^n - 1)$

FACTORISE $= P(1.005)^n - 500000(1.005^n) + 500\,000$

$= (P - 500\,000) \times 1.005^n + 500\,000$

β. $0 = (P - 500\,000) \times 1.005^n + 500\,000$

$-500\,000 = (P - 500\,000) \times 1.005^n$

$\dfrac{-500\,000}{P - 500\,000} = 1.005^n$

(= 313 406.52 FROM i.)

$2.6796\ldots = 1.005^n$

$\dfrac{\ln(2.6796)}{\ln(1.005)} = n$

$\therefore n = 197.62$ MONTHS

$= 197$ ~~or~~ 198.

QUESTION 18.

ai. $\int 3e^t + 6e^{-t}$

$= 3e^t - 6e^{-t} + C$

$t = 0, x = 0$: $0 = 3 - 6 + C$

$0 = -3 + C$

$\therefore C = 3$

$\therefore x = 3e^t - 6e^{-t} + 3$

ii. $10 = 3e^t - 6e^{-t} + 3$

$7 = 3e^t - 6e^{-t}$

$7 = 3e^t - \dfrac{6}{e^t}$ $\quad \times e^t$

$7e^t = 3e^{2t} - 6$

$0 = 3e^{2t} - 7e^t - 6$ PTO

Let $u = e^t$

$0 = 3u^2 - 7u - 6$

$0 = (3u+2)(u-3)$

$\therefore u = -2/3, 3$

$e^t \neq -2/3$ NO SOL'N

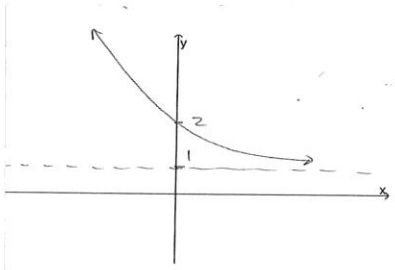
$e^t = 3$

$\frac{\ln 3}{\ln e} = t$

$\therefore t = \ln 3$ secs

bi. $y = e^{-x} + 1$

ii.



iii. $y = e^{-\ln x} + 1$

$y = \frac{1}{x} + 1$

D: All real x, $x > 0$

R: All real y, $y > 1$

QUESTION 19.

ai.

8	9	10	11	12
2,6	3,6	4,6	5,6	6,6
3,5	4,5	5,5	6,5	
4,4	5,4	6,4		
5,3	6,3			
6,2				

ii. $\therefore P(\text{EVEN} \mid \text{SUM} > 7) = \frac{9}{36} = \frac{1}{4}$

(36 OUTCOMES. 6×6)

P(AT LEAST 1 EVEN)

$= 1 - P(\text{NOT EVEN})$

$= 1 - (3/6 \times 3/6)$

$= 3/4$

· OF 36 OUTCOMES $3/4 \times 36 = 27$ AT LEAST 1 EVEN.

· FROM 9 OUTCOMES IN (i) 6 HAVE AT LEAST 1 EVEN.

$\therefore 6/27$

bi. $\cos\theta = \frac{BD}{a}$

$\therefore BD = a\cos\theta$

$\sin\theta = \frac{CD}{a}$

$\therefore CD = a\sin\theta$.

ii. ADDING UP ALL SIDES...

$L = 4 + 2a\cos\theta + a\sin\theta + a$.

PTO

iii. $L = 4 + 6\sqrt{5}\cos\theta + 3\sqrt{5}\sin\theta + 3\sqrt{5}$

$L' = -6\sqrt{5}\sin\theta + 3\sqrt{5}\cos\theta$

STAT. PTS: $0 = -6\sqrt{5}\sin\theta + 3\sqrt{5}\cos\theta$

$6\sqrt{5}\sin\theta = 3\sqrt{5}\cos\theta$

$6\sqrt{5}\dfrac{\sin\theta}{\cos\theta} = 3\sqrt{5}$

$\tan\theta = \dfrac{3\sqrt{5}}{6\sqrt{5}}$

$\tan\theta = 1/2$ DON'T USE CALC, USE $\triangle$:

$\therefore \sin\theta = \dfrac{1}{\sqrt{5}}$ & $\cos\theta = \dfrac{2}{\sqrt{5}}$

TEST NATURE:

$L'' = -6\sqrt{5}\cos\theta - 3\sqrt{5}\sin\theta$

$= -6\sqrt{5} \times \dfrac{2}{\sqrt{5}} - 3\sqrt{5} \times \dfrac{1}{\sqrt{5}}$

$= -12 - 3 = -15$ -VE $\cap$ $\therefore$ MAX

$\therefore$ MAX LENGTH:

$L = 4 + \left(6\sqrt{5} \times \dfrac{2}{\sqrt{5}}\right) + \left(3\sqrt{5} \times \dfrac{1}{\sqrt{5}}\right) + 3\sqrt{5}$

$= 4 + 12 + 3 + 3\sqrt{5}$

$= 19 + 3\sqrt{5}$ cm. DONE!